GCSE Maths – Probability

Enumeration, Venn Diagrams, Tree Diagrams and Tables

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of enumeration questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Jason has 4 counters in a bag. One counter is red (R), one is blue (B), one is green (G) and one is yellow (Y). He draws the counters from the bag one at a time. In how many different orders could he draw the counters?

Step 1: Use systematic listing to work through each combination.

Start by fixing one of the items in position 1 and another in position 2. Then swap the last two items in positions 3 and 4 to produce the first two combinations. Then, change the second item and swap the last two items again. Continue until you have found all combinations relating to the fixed first position item.

Y G B R
Y G R B
Y R G B
Y R B G
Y B G R
Y B R G

Step 2: Consider another item in position 1. Work through each possibility, swapping the numbers until you have tried every combination of numbers in every position.

G Y R B	*R G Y B*	*B G R Y*
G Y B R	*R G B Y*	*B G Y R*
G B Y R	*R B Y G*	*B Y R G*
G B R Y	*R B G Y*	*B Y G R*
G R B Y	*R Y G B*	*B R G Y*
G R Y B	*R Y B G*	*B R Y G*

Step 3: Count the number of combinations you have found. Double check that you have not repeated any combinations.

There are 6 combinations in the first column. You could use this, and the fact that there are 4 colours of counter, to work out that the total number of combinations is:

$$6 \times 4 \; = \; 24 \; combinations \; of \; counters$$

Guided Example

In a sandwich shop there are 2 kinds of bread, 3 fillings and 4 drinks to choose from. How many different meal deals can be made?

Step 1: List or calculate the possible combinations.

You could use the product rule for this question because there are a lot of variables to consider listing.

Product rule : multiply the number of items in each group together

number of bread × number of fillings × number of drinks

2 × 3 × 4

24 possible combinations

Alternatively, you can list the possible combinations using a table.

	Bread 1 (B1)	Bread 2 (B2)
Filling 1 (F1)	B1, F1	B2, F1
Filling 2 (F2)	B1, F2	B2, F2
Filling 3 (F3)	B1, F3	B2, F3

use previous combinations with 4 different drinks

	Drink 1 (D1)	Drink 2 (D2)	Drink 3 (D3)	Drink 4 (D4)
B1, F1	B1, F1, D1	B1, F1, D2	B1, F1, D3	B1, F1, D4
B1, F2	B1, F2, D1	B1, F2, D2	B1, F2, D3	B1, F2, D4
B1, F3	B1, F3, D1	B1, F3, D2	B1, F3, D3	B1, F3, D4
B2, F1	B2, F1, D1	B2, F1, D2	B2, F1, D3	B2, F1, D4
B2, F2	B2, F2, D1	B2, F2, D2	B2, F2, D3	B2, F2, D4
B2, F3	B2, F3, D1	B2, F3, D2	B2, F3, D3	B2, F3, D4

24 possible combinations

count the new combinations

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Janae has 4 shirts, 2 pairs of jeans and 2 pairs of shoes. How many outfits can she make?

Product rule : shirts × jeans × shoes
: 4 × 2 × 2
: 16

There are 16 possible outfits

Alternative method :

①

	Jeans 1 (J1)	Jeans 2 (J2)
Shoes 1 (S1)	S1, J1	S1, J2
Shoes 2 (S2)	S2, J1	S2, J2

②

16 combinations

	S1, J1	S2, J1	S1, J2	S2, J2
Shirt 1 (H1)	H1, S1, J1	H1, S2, J1	H1, S1, J2	H1, S2, J2
Shirt 2 (H2)	H2, S1, J1	H2, S2, J1	H2, S1, J2	H2, S2, J2
Shirt 3 (H3)	H3, S1, J1	H3, S2, J1	H3, S1, J2	H3, S2, J2
Shirt 4 (H4)	H4, S1, J1	H4, S2, J1	H4, S1, J2	H4, S2, J2

2. Louis is going to have a meal. He can choose one main and one dessert from the menu. Write down all the possible combinations that Louis could choose.

Menu	
Main	**Dessert**
Soup	Ice Cream
Pasta	Cheesecake
Pizza	Tiramisu

Dessert

Main	Ice cream	Cheesecake	Tiramisu
Soup	Ice cream, soup	Cheesecake, Soup	Tiramisu, Soup
Pasta	Icecream, pasta	Cheesecake, Pasta	Tiramisu, Pasta
Pizza	Ice cream, Pizza	Cheesecake, Pizza	Tiramisu, Pizza

Possible combinations :

① Ice cream, soup
② Ice cream, pasta
③ Ice cream, pizza
④ Cheesecake, soup
⑤ Cheesecake, pasta
⑥ Cheesecake, pizza
⑦ Tiramisu, soup
⑧ Tiramisu, pasta
⑨ Tiramisu, pizza

3. Maia flips a coin and rolls a 6-sided die. Use the following table to help list all the possible outcomes.

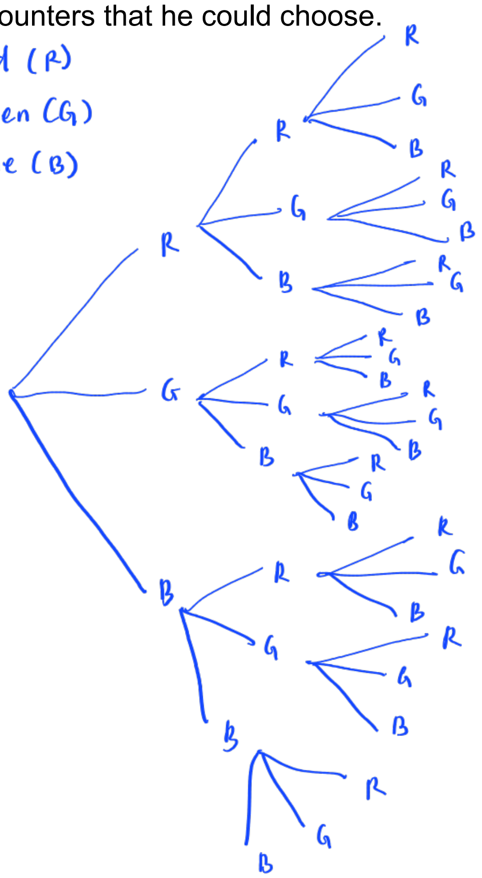
		Die					
		1	2	3	4	5	6
Coin	Heads	Heads, 1	Heads, 2	Heads, 3	Heads, 4	Heads, 5	Heads, 6
	Tails	Tails, 1	Tails, 2	Tails, 3	Tails, 4	Tails, 5	Tails, 6

List of possible outcomes:

1. Heads with 1
2. Heads with 2
3. Heads with 3
4. Heads with 4
5. Heads with 5
6. Heads with 6
7. Tails with 1
8. Tails with 2
9. Tails with 3
10. Tails with 4
11. Tails with 5
12. Tails with 6

4. Justin has a bag containing red, green and blue counters. He picks three counters from the bag one at a time. Draw a tree diagram to show the possible combinations of counters that he could choose.

Red (R)
Green (G)
Blue (B)



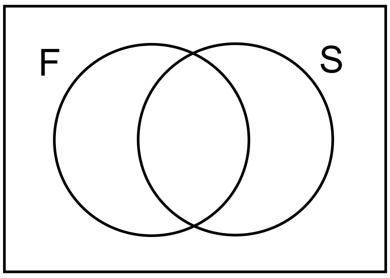
Possible combinations of counters:

1. RRR
2. RRG
3. RRB
4. RGR
5. RGG
6. RGB
7. RBR
8. RBG
9. RBB
10. GRR
11. GRG
12. GRB
13. GGR
14. GGG
15. GGB
16. GBR
17. GBG
18. GBB
19. BRR
20. BRG
21. BRB
22. BGR
23. BGG
24. BGB
25. BBR
26. BBG
27. BBB

Section B

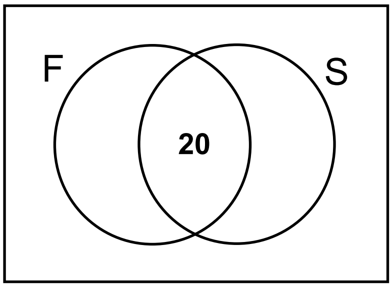
Worked Example

A school has 200 students. There are 80 students in the French class (F). There are 60 students in the Spanish class (S). Twenty students study both languages. Complete the Venn diagram to show this information.



Step 1: Work out how many students study both languages and write this in the intersection of the circles.

20 students study both languages.



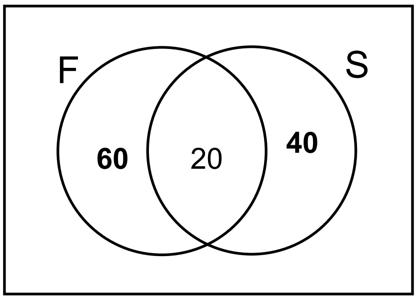
Step 2: Calculate the number of students who study just French or just Spanish.

80 students study French, but 20 of them study French AND Spanish:

$$80 - 20 = 60 \textit{ students study just French.}$$

60 students study Spanish, but 20 of them study French AND Spanish:

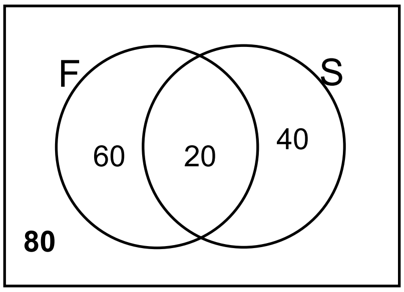
$$60 - 20 = 40 \textit{ students study just Spanish.}$$



Step 3: Calculate the number of students who do not study a language. Write this number outside of the circles.

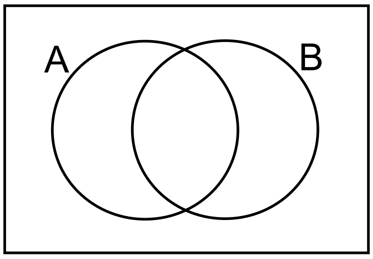
$$60 + 20 + 40 = 120 \textit{ students study a language}$$

$$200 - 120 = 80 \textit{ students do not study a language}$$



Guided Example

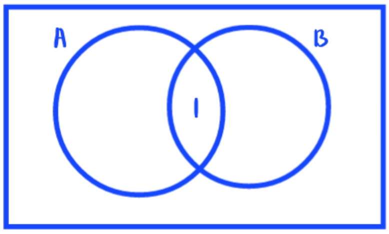
Consider the set of data values
$D = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$.
Let $A = multiples\ of\ 3$
Let $B = multiples\ of\ 4$.
Complete the Venn diagram for the set D.



Step 1: For each number in the set, decide which numbers are a multiple of 3 **and** a multiple of 4. Write these numbers in the intersection of A and B.

A (multiples of 3) : {3, 6, 9, 12, 15, 18} = 6 numbers
B (multiples of 4) : {4, 8, 12, 16} = 4 numbers
$(A \cap B)$: {12} = 1 number

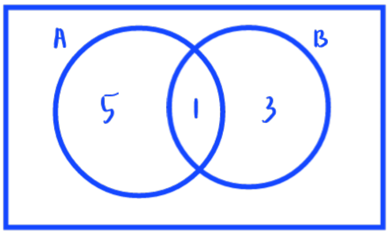
find a common number in data set A and B



Step 2: Write the remaining multiples of 3 and 4 in the correct circles.

$(A \cap B)$
A = 6 − 1 = 5
B = 4 − 1 = 3

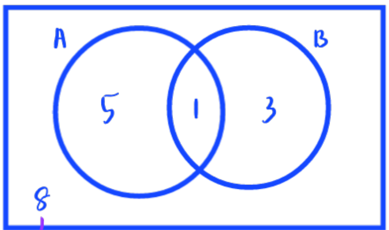
multiple of both 3 and 4



Step 3: Write the numbers that are neither multiples of 3 nor 4 in the correct section of the Venn diagram.

D = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} = 17 numbers

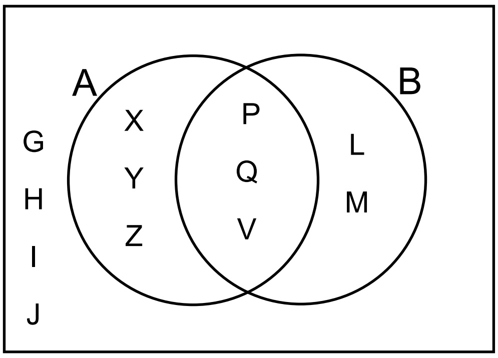
Remaining numbers that are neither multiples of 3 nor 4 = 17 − 1 − 5 − 3
= 8



write the remaining number inside the box but outside the circles

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Here is a Venn diagram. Write down the letters that are in:



a) A

$A = \{P, Q, V, X, Y, Z\}$

b) A' → not inside A

$A' = \{G, H, I, J, L, M\}$

c) $A \cap B$ → inside both A and B

$(A \cap B) = \{P, Q, V\}$

6. There are 60 members in an athletics club.

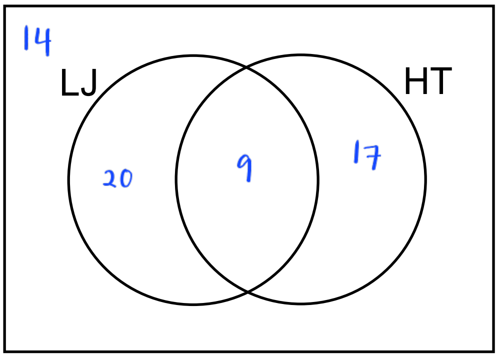
9 athletes do long jump and hammer throw.
20 athletes do only long jump.
14 athletes do not do long jump or hammer throw.

Complete the Venn diagram. How many athletes only do hammer throw?

athletes only do hammer throw :

$= 60 - 9 - 20 - 14$

$= 17$

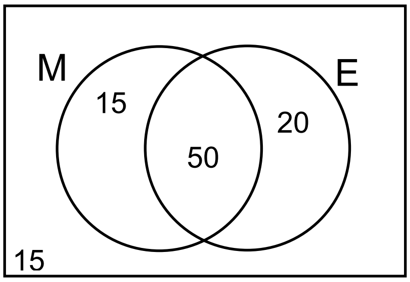


There are 17 athletes that only do hammer throw.

Section C

Worked Example

The Venn diagram below shows whether students passed their Maths and English exams. A student is chosen at random. Find the probability that they passed Maths and English.



Step 1: Find the population total.

There are 15 students who did not pass either exam and 50 students who passed both. There are also 15 students who passed only Maths, and 20 students who passed only English.

$$15 + 50 + 15 + 20 = 100\ students$$

Step 2: Find the number of students in the target space.

We are trying to find $P(M \cap E)$*. This means we need to look at the intersection between Maths and English.*

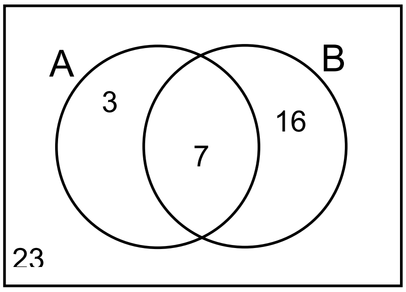
There are 50 students in the intersection.

Step 3: Calculate the probability.

$$Probability(M \cap E\,) = \frac{50}{100} = \frac{1}{2}$$

Guided Example

Using the Venn Diagram, find P(A').



Step 1: Find the population total.

population total : 23 + 7 + 3 + 16 = 49

Step 2: Find the number in the target space A'. → not in A

A' = 23 + 16 = 39

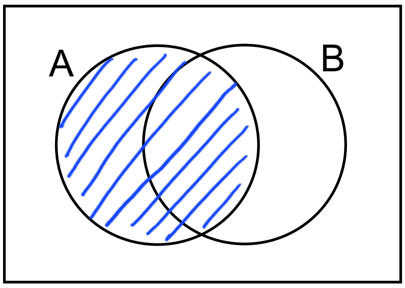
Step 3: Calculate the required probability.

$P(A') = \frac{39}{49}$

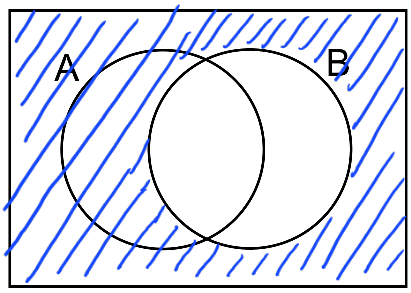
Now it's your turn!
If you get stuck, look back at the worked and guided examples.

7. Shade the given areas:

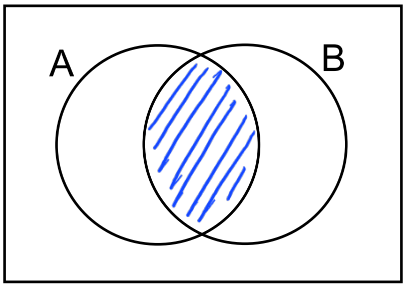
a) A



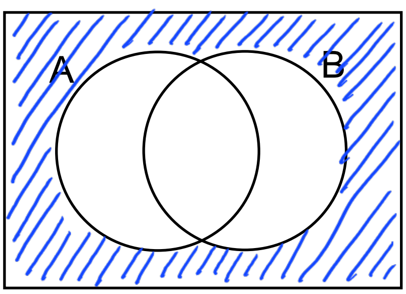
b) B'



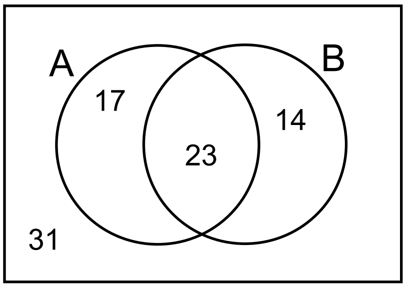
c) A ∩ B



d) A' ∩ B'



8. From the Venn diagram, find the following probabilities:



a) P(A)

$A = 17 + 23 = 40$

Total population : $17 + 31 + 23 + 14 = 85$

$P(A) = \frac{40 \div 5}{85 \div 5} = \frac{8}{17}$

b) P(B')

$B' = 17 + 31 = 48$

Total population $= 31 + 17 + 23 + 14 = 85$

$P(B') = \frac{48}{85}$

c) $P(A \cap B)$

$A \cap B = 23$

Total population $= 85$

$P(A \cap B) = \frac{23}{85}$

d) $P(A' \cup B)$ → in combination with

$A' \cup B = 14 + 31 + 23 = 68$

Total population $= 85$

$P(A' \cup B) = \frac{68 \div 17}{85 \div 17} = \frac{4}{5}$

e) $P((A \cap B)')$

$(A \cap B)' = 31 + 17 + 14 = 62$

Total population $= 85$

$P((A \cap B)') = \frac{62}{85}$

9. Juliana goes to the local leisure centre. She records that 50 people visit that day. Of those that visit,

30 visited the gym
13 people went swimming
22 people went to the spa
6 people went swimming and went to the spa
11 people went to the spa and visited the gym
12 people went swimming and visited the gym
5 people went swimming, visited the gym and went to the spa

a) Draw a Venn diagram to display this information.

people who goes to gym only :

$30 - 6 - 5 - 7 = 12$

people who goes to spa only :

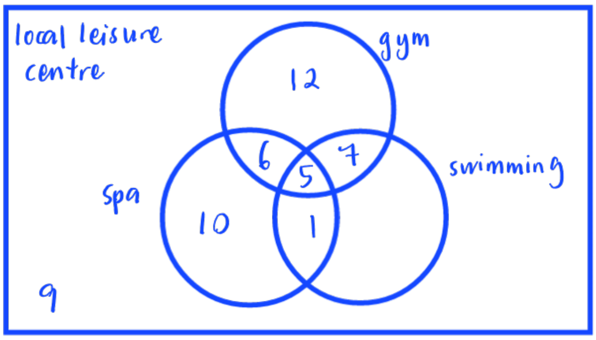
$22 - 6 - 5 - 1 = 10$

people who goes swimming only :

$13 - 7 - 5 - 1 = 0$

people who didn't go to any :

$50 - 12 - 6 - 5 - 7 - 10 - 1 = 9$



b) How many people visited the leisure centre but did not visit any of the gym, spar or swimming pool?

People who visited the leisure centre but did not visit any of the gym, spa or swimming pool :

$50 - 10 - 6 - 5 - 1 - 7 - 12 = 9$

There are 9 people.

c) One person is picked at. What is the probability that they visited the gym and went swimming?

People who went to gym and swimming :

$(\text{Gym} \cap \text{swimming}) = 12$

Total population $= 50$

$P(\text{gym} \cap \text{swimming}) = \frac{12 \div 2}{50 \div 2} = \frac{6}{25}$